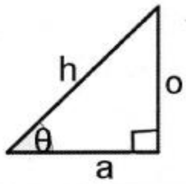
MODULE 7: Trigonometry

Revision

$\sin\theta = \frac{o}{h}$

$\cos\theta = \frac{a}{h}$

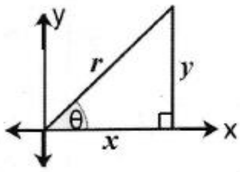
$\tan\theta = \frac{o}{a}$

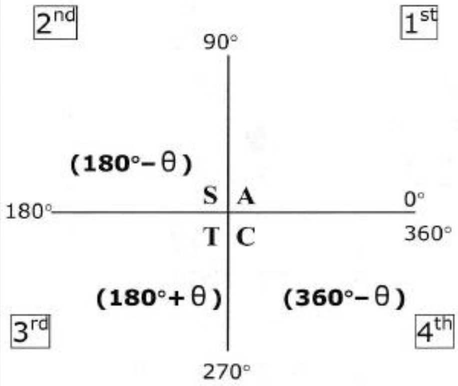


$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$





Second quadrant

$\sin(180°-\theta)=\sin\theta$

$\cos(180°-\theta)=-\cos\theta$

$\tan(180°-\theta)=-\tan\theta$

Third quadrant

$\tan(180°+\theta)=\tan\theta$

$\cos(180°+\theta)=-\cos\theta$

$\sin(180°+\theta)=-\sin\theta$

Fourth quadrant

$\cos(360°-\theta)=\cos\theta$

$\sin(360°-\theta)=-\sin\theta$

$\tan(360°-\theta)=-\tan\theta$

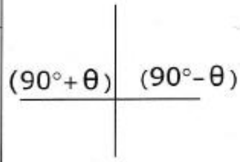
Co-functions

$\sin(90°-\theta)=\cos\theta$

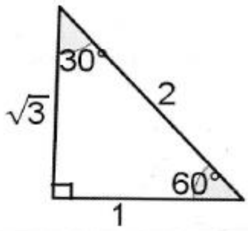
$\cos(90°-\theta)=\sin\theta$

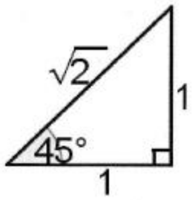
$\sin(90°+\theta)=\cos\theta$

$\cos(90°+\theta)=-\sin\theta$



Special Traingles **KNOW!**





Negative angles

$\cos(-A) = \cos A$

$\sin(-A) = -\sin A$

$\tan(-A) = -\tan A$

Study the following theory well.

Compound-angle identities

$\cos(A - B) = \cos A\cos B + \sin A\sin B$

$\cos(A + B) = \cos A\cos B - \sin A\sin B$

$\sin(A + B) = \sin A\cos B + \cos A\sin B$

$\sin(A - B) = \sin A\cos B - \cos A\sin B$

Double-angle identities

$\sin 2A = 2\sin A\cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$= 1 - 2\sin^2 A$

$= 2\cos^2 A - 1$

$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

Identities

$\sin^2 A + \cos^2 A = 1$

$\therefore \cos^2 A = 1 - \sin^2 A$

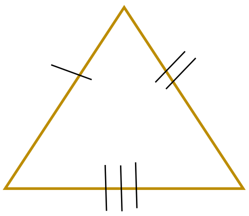
and $\sin^2 A = 1 - \cos^2 A$

$\tan A = \frac{\sin A}{\cos A}$

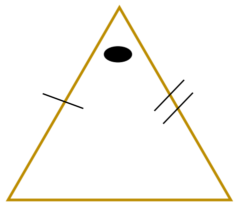
Negtive angle (Add 360°) 1. sin(–120°) = sin 240° = sin(180°+60°) = – sin 60° $= \frac{-\sqrt{3}}{2}$	**Angle greater than 360° (subtract 360°)** 2. tan 420° = tan60° $= \frac{\sqrt{3}}{1}$
Not a reduction formulae (+ or – 360°) 1. sin (540°– θ) = sin(180° – θ) = sin θ 2. tan (θ – 360°) = tan θ 3. cos $(\theta - 180°)$ = cos $(\theta + 180°)$ = – cos θ	**Exception** $(\theta - 90°)$ (take out a negative) 1. sin $(\theta - 90°)$ = sin – (90°– θ) = – sin (90°– θ) = – cos θ
Square **(square goes outside the brackets)** 1. $\sin^2(180° - \theta)$ $= [\sin(180° - \theta)]^2$ $= [-\sin\theta]^2$ $= \sin^2\theta$	**Cofunctions are equal if their angles add up to 90°** 1. sin 30° = cos 60° 2. cos 20° = sin 70°

USE COSINE WHEN YOU ARE GIVEN:

1. SSS

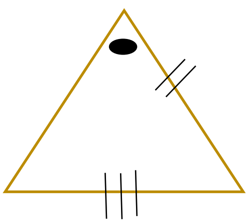


2. SAS (TWO SIDES AND AN INCLUDED ANGLE)

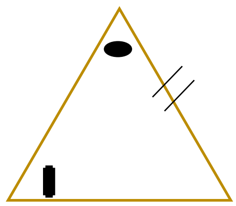


USE SINE WHEN YOU ARE GIVEN:

1. SSA



2. AAS



TRIG NOTES 14/03/2026

EXAMPLE

5.3 Given the expression:

$$\frac{\sin(90°+x).\cos(-x).\tan^2(540°+x)}{\cos(180°-x).\sin(x-90°)-1}$$

5.3.1 Simplify the expression fully. (5)

5.3.2 For which value(s) of x, is the identity undefined? (3)

5.3.1	$=\frac{(\cos x)(\cos x)(\tan^2 x)}{(-\cos x)(-cox)-1}$ $=\frac{\cos^2 x.\tan^2 x}{\cos^2 x-1}$ $=\frac{\cos^2 x.\frac{\sin^2 x}{\cos^2 x}}{\cos^2 x-1}$ $=\frac{\sin^2 x}{-\sin^2 x}$ $=-1$	✓ $\cos x.\cos x$ ✓ $\tan^2 x$ ✓ $-\cos x.-\cos x$ ✓ $-\sin^2 x$ ✓ -1 (5)
5.3.2	$\cos^2 x-1=0$ $\cos x = \pm 1$ $ref\ angle = 0°$ $\therefore x=0°$ or $x=180°$ or $x=360°$	✓ $x=0°$ ✓ $x=180°$ ✓ $x=360°$ (3)

EXAMPLE

If $\sin 14° = k$, write the following in terms of k:

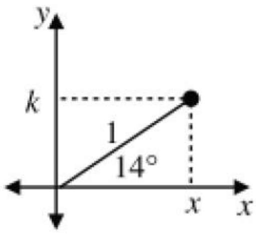
(a)	$\sin 194°$	(b)	$\cos 76°$	(c)	$\cos 14°$
(d)	$\sin 28°$	(e)	$\cos 28°$	(f)	$\tan 28°$
(g)	$\cos 44°$	(h)	$\sin 7°\cos 7°$	(i)	$1-2\sin^2 7°$

Solution

(a)
$$\sin 194^\circ$$
$$= \sin(180 + 14^\circ)$$
$$= -\sin 14^\circ$$
$$= -k$$

(b)
$$\cos 76^\circ$$
$$= \cos(90^\circ - 14^\circ)$$
$$= \sin 14^\circ$$
$$= k$$

(c) $\sin 14^\circ = \frac{k}{1} \quad \left(\frac{y}{r}\right)$



$$x^2 + y^2 = r^2$$
$$\therefore x^2 + k^2 = 1^2$$
$$\therefore x^2 = 1 - k^2$$
$$\therefore x = \sqrt{1-k^2}$$

$x = \sqrt{1-k^2}$

$y = k$

$r = 1$

$$\cos 14^\circ = \frac{x}{r} = \frac{\sqrt{1-k^2}}{1} = \sqrt{1-k^2}$$

(d)
$$\sin 28^\circ$$
$$= \sin(2 \cdot 14^\circ)$$
$$= 2\sin 14^\circ \cos 14^\circ$$
$$= 2k\sqrt{1-k^2}$$

(e)
$$\cos 28^\circ$$
$$= \cos(2 \cdot 14^\circ)$$
$$= 1 - 2\sin^2 14^\circ$$
$$= 1 - 2k^2$$

(f)
$$\tan 28^\circ$$
$$= \frac{\sin 28^\circ}{\cos 28^\circ}$$
$$= \frac{2k\sqrt{1-k^2}}{1-2k^2}$$

(g)
$$\cos 44^\circ$$
$$= \cos(30^\circ + 14^\circ)$$
$$= \cos 30^\circ \cos 14^\circ - \sin 30^\circ \sin 14^\circ$$
$$= \frac{\sqrt{3}}{2} \cdot \sqrt{1-k^2} - \frac{1}{2}k$$
$$= \frac{\sqrt{3-3k^2} - k}{2}$$

(h)
$$\sin 7^\circ \cos 7^\circ$$
$$= \frac{1}{2}(2\sin 7^\circ \cos 7^\circ)$$
$$= \frac{1}{2}\sin 14^\circ$$
$$= \frac{1}{2}k$$

(i)
$$1 - 2\sin^2 7^\circ$$
$$= \cos 14^\circ$$
$$= \sqrt{1-k^2}$$

DO THIS EXAMPLE

$$\frac{2\sin 510^\circ - \cos 340^\circ \cdot \cos(-20^\circ)}{\cos^2 110^\circ}$$

EXAMPLE 6

Simplify the following:

(a) $\dfrac{\sin 2\theta}{\sin\theta}$ (b) $\dfrac{\cos 2\alpha}{\cos\alpha - \sin\alpha}$

(c) $\dfrac{\cos 2A + 1}{2\cos A}$ (d) $\cos 2x + 2\sin^2 x$

Solution

(a) $\dfrac{\sin 2\theta}{\sin\theta}$

$= \dfrac{2\sin\theta\cos\theta}{\sin\theta}$

$= 2\cos\theta$

(b) $\dfrac{\cos 2\alpha}{\cos\alpha - \sin\alpha}$

$= \dfrac{\cos^2\alpha - \sin^2\alpha}{\cos\alpha - \sin\alpha}$

$= \dfrac{(\cos\alpha - \sin\alpha)(\cos\alpha + \sin\alpha)}{\cos\alpha - \sin\alpha}$

$= \cos\alpha + \sin\alpha$

(c) $\dfrac{\cos 2A + 1}{2\cos A}$

$= \dfrac{2\cos^2 A - 1 + 1}{2\cos A}$

$= \dfrac{2\cos^2 A}{2\cos A}$

$= \cos A$

(d) $\cos 2x + 2\sin^2 x$

$= 1 - 2\sin^2 x + 2\sin^2 x$

$= 1$

EXAMPLE 20

Prove that: $\tan\theta + \dfrac{\cos\theta}{\sin\theta} = \dfrac{2}{\sin 2\theta}$

Solution

$\text{LHS} = \tan\theta + \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$

$= \dfrac{1}{\cos\theta\sin\theta}$

$\text{RHS} = \dfrac{2}{\sin 2\theta}$

$= \dfrac{2}{2\sin\theta\cos\theta}$

$= \dfrac{1}{\cos\theta\sin\theta}$

$\therefore \text{LHS} = \text{RHS}$

EXAMPLE 21

Prove that:

(a) $$\frac{\sin 2A}{1+\cos 2A}=\tan A$$

(b) $$\frac{1-\cos 2x}{\sin x}=\frac{\sin 2x}{\cos x}$$

(c) $$\frac{\cos 2\theta}{\cos\theta-\sin\theta}=\frac{\sin\theta}{\tan\theta}+\tan\theta\cos\theta$$

Solution

(a) $$\text{LHS}=\frac{\sin 2A}{1+\cos 2A}$$

$$=\frac{2\sin A\cos A}{1+2\cos^2 A-1}$$

$$=\frac{2\sin A\cos A}{2\cos^2 A}$$

$$=\frac{\sin A}{\cos A}$$

$$\text{RHS}=\tan A$$

$$=\frac{\sin A}{\cos A}$$

(b) $$\text{LHS}=\frac{1-\cos 2x}{\sin x}$$

$$=\frac{1-(1-2\sin^2 x)}{\sin x}$$

$$=\frac{2\sin^2 x}{\sin x}$$

$$=2\sin x$$

$$\text{RHS}=\frac{\sin 2x}{\cos x}$$

$$=\frac{2\sin x\cos x}{\cos x}$$

$$=2\sin x$$

(c) $$\text{LHS}=\frac{\cos 2\theta}{\cos\theta-\sin\theta}$$

$$=\frac{\cos^2\theta-\sin^2\theta}{\cos\theta-\sin\theta}$$

$$=\frac{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}{\cos\theta-\sin\theta}$$

$$=\cos\theta+\sin\theta$$

$$\text{RHS}=\frac{\sin\theta}{\tan\theta}+\tan\theta\cos\theta$$

$$=\frac{\sin\theta}{\left(\frac{\sin\theta}{\cos\theta}\right)}+\left(\frac{\sin\theta}{\cos\theta}\right)\cos\theta$$

$$=\frac{\sin\theta}{1}\times\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}\times\frac{\cos\theta}{1}$$

$$=\cos\theta+\sin\theta$$

EXAMPLE 24

Prove that:

(a) $\sin(A+B)+\sin(A-B)=2\sin A\cos B$ (b) $\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$

Solution

(a) $\text{LHS} = \sin(A+B)+\sin(A-B)$ $\text{RHS} = 2\sin A\cos B$

$= \sin A\cos B + \cos A\sin B + \sin A\cos B - \cos A\sin B$

$= 2\sin A\cos B$

(b) $\text{LHS} = \cos 3\alpha$ $\text{RHS} = 4\cos^3\alpha - 3\cos\alpha$

$= \cos(2\alpha+\alpha)$

$= \cos 2\alpha\cos\alpha - \sin 2\alpha\sin\alpha$

$= (2\cos^2\alpha - 1)\cos\alpha - (2\sin\alpha\cos\alpha)\sin\alpha$

$= 2\cos^3\alpha - \cos\alpha - 2\sin^2\alpha\cos\alpha$

$= 2\cos^3\alpha - \cos\alpha - 2(1-\cos^2\alpha)\cos\alpha$

$= 2\cos^3\alpha - \cos\alpha - 2\cos\alpha + 2\cos^3\alpha$

$= 4\cos^3\alpha - 3\cos\alpha$

EXAMPLE 25

Prove that: $\dfrac{\sin 3x\cos x - \cos 3x\sin x}{\cos^2 x - \sin^2 x} = \tan 2x$

Solution

$\text{LHS} = \dfrac{\sin(3x-x)}{\cos 2x}$ $\text{RHS} = \tan 2x$

$= \dfrac{\sin 2x}{\cos 2x}$ $= \dfrac{\sin 2x}{\cos 2x}$

EXAMPLE 26

Prove that: $\dfrac{\sin 3x + \sin 7x}{\cos 3x + \cos 7x} = \tan 5x$

Solution

Notice that the average of $3x$ and $7x$ is $5x$. $\left(\dfrac{3x+7x}{2} = \dfrac{10x}{2} = 5x\right)$

We can write:

- $3x$ as $5x - 2x$
- $7x$ as $5x + 2x$

$$\begin{aligned}
\text{LHS} &= \frac{\sin 3x + \sin 7x}{\cos 3x + \cos 7x} \\
&= \frac{\sin(5x-2x)+\sin(5x+2x)}{\cos(5x-2x)+\cos(5x+2x)} \\
&= \frac{\sin 5x\cos 2x - \cos 5x\sin 2x + \sin 5x\cos 2x + \cos 5x\sin 2x}{\cos 5x\cos 2x + \sin 5x\sin 2x + \cos 5x\cos 2x - \sin 5x\sin 2x} \\
&= \frac{2\sin 5x\cos 2x}{2\cos 5x\cos 2x} \\
&= \frac{\sin 5x}{\cos 5x}
\end{aligned}$$

$$\begin{aligned}
\text{RHS} &= \tan 5x \\
&= \frac{\sin 5x}{\cos 5x}
\end{aligned}$$

EXAMPLE 23

Prove that: $\dfrac{1-\sin 2x}{\cos x - \sin x} = \dfrac{\cos 2x}{\cos x + \sin x}$

Solution

$$\begin{aligned}
\text{LHS} &= \frac{1-\sin 2x}{\cos x - \sin x} \\
&= \frac{1-2\sin x\cos x}{\cos x - \sin x} \\
&= \frac{\cos^2 x + \sin^2 x - 2\sin x\cos x}{\cos x - \sin x} \\
&= \frac{\cos^2 x - 2\sin x\cos x + \sin^2 x}{\cos x - \sin x} \\
&= \frac{(\cos x - \sin x)^2}{\cos x - \sin x} \\
&= \cos x - \sin x
\end{aligned}$$

$$\begin{aligned}
\text{RHS} &= \frac{\cos 2x}{\cos x + \sin x} \\
&= \frac{\cos^2 x - \sin^2 x}{\cos x + \sin x} \\
&= \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x + \sin x} \\
&= \cos x - \sin x
\end{aligned}$$

EXAMPLE 32

Solve for θ:

(a) $2\sin 2\theta = 3\cos\theta$ (b) $3\sin\theta - \cos 2\theta + 2 = 0$

(c) $\cos 2\theta + 2\sin 2\theta - 4\sin^2\theta = 0$

Solution

(a) $2\sin 2\theta = 3\cos\theta$

Replace $\sin 2\theta$ with $2\sin\theta\cos\theta$:

$\therefore 2(2\sin\theta\cos\theta) = 3\cos\theta$

$\therefore 4\sin\theta\cos\theta = 3\cos\theta$

$\therefore 4\sin\theta\cos\theta - 3\cos\theta = 0$

$\therefore \cos\theta(4\sin\theta - 3) = 0$

$\therefore \cos\theta = 0$ or $4\sin\theta - 3 = 0$

$\therefore \cos\theta = 0$ or $\sin\theta = \frac{3}{4}$

- $\cos\theta = 0$: $\theta = 90° + k\cdot 360°$ or $\theta = 270° + k\cdot 360°$; $k \in \mathbb{Z}$
- $\sin\theta = \frac{3}{4}$: $\theta = 48{,}59° + k\cdot 360°$ or $\theta = 131{,}41° + k\cdot 360°$; $k \in \mathbb{Z}$

(b) $3\sin\theta - \cos 2\theta + 2 = 0$

$\sin\theta$ in the equation $\rightarrow$ Replace $\cos 2\theta$ with $1 - 2\sin^2\theta$:

$3\sin\theta - (1 - 2\sin^2\theta) + 2 = 0$

$\therefore 3\sin\theta - 1 + 2\sin^2\theta + 2 = 0$

$\therefore 2\sin^2\theta + 3\sin\theta + 1 = 0$

$\therefore (2\sin\theta + 1)(\sin\theta + 1) = 0$

$\therefore \sin\theta = -\frac{1}{2}$ or $\sin\theta = -1$

- $\sin\theta = -\frac{1}{2}$: $\theta = 210° + k\cdot 360°$ or $\theta = 330° + k\cdot 360°$; $k \in \mathbb{Z}$
- $\sin\theta = -1$: $\theta = 270° + k\cdot 360°$; $k \in \mathbb{Z}$

EXAMPLE 33

Solve for α:

(a) $\cos\alpha\cos 10° + \sin\alpha\sin 10° = 0,365$ (b) $\sin\alpha\cos\alpha = -0,4$

Solution

(a) $\cos\alpha\cos 10° + \sin\alpha\sin 10° = 0,365$

Do you recognise the compound angle pattern? $\cos\alpha\cos 10° + \sin\alpha\sin 10° = \cos(\alpha - 10°)$:

$\therefore \cos(\alpha - 10°) = 0,365$

$\therefore \alpha - 10° = 68,59° + k \cdot 360°$ or $\alpha - 10° = 291,41° + k \cdot 360°$; $k \in \mathbb{Z}$

$\therefore \alpha = 78,59° + k \cdot 360°$ or $\alpha = 301,41° + k \cdot 360°$; $k \in \mathbb{Z}$

(b) $\sin\alpha\cos\alpha = -0,4$

Multiply both sides by 2 to create $2\sin\alpha\cos\alpha$, which can be replaced with $\sin 2\alpha$:

$2\sin\alpha\cos\alpha = -0,8$

$\therefore \sin 2\alpha = -0,8$

$\therefore 2\alpha = 233,13° + k \cdot 360°$ or $2\alpha = 306,87° + k \cdot 360°$; $k \in \mathbb{Z}$

$\therefore \alpha = 116,57° + k \cdot 180°$ or $\alpha = 153,44° + k \cdot 180°$; $k \in \mathbb{Z}$

(c) $\cos 2\theta + 2\sin 2\theta - 4\sin^2\theta = 0$

Replace $\sin 2\theta$ with $2\sin\theta\cos\theta$:

$\cos 2\theta + 2(2\sin\theta\cos\theta) - 4\sin^2\theta = 0$

$\therefore \cos 2\theta + 4\sin\theta\cos\theta - 4\sin^2\theta = 0$

$\sin\theta\cos\theta$ in the equation $\rightarrow$ Replace $\cos 2\theta$ with $\cos^2\theta - \sin^2\theta$:

$\therefore \cos^2\theta - \sin^2\theta + 4\sin\theta\cos\theta - 4\sin^2\theta = 0$

$\therefore \cos^2\theta + 4\sin\theta\cos\theta - 5\sin^2\theta = 0$

$\therefore (\cos\theta - \sin\theta)(\cos\theta + 5\sin\theta) = 0$

$\therefore \cos\theta = \sin\theta$ or $\cos\theta = -5\sin\theta$

$\therefore \tan\theta = 1$ or $\tan\theta = -\frac{1}{5}$

- $\tan\theta = 1$: $\theta = 45° + k \cdot 360°$ or $\theta = 225° + k \cdot 360°$; $k \in \mathbb{Z}$
- $\tan\theta = -\frac{1}{5}$: $\theta = 168,69° + k \cdot 360°$ or $\theta = 348,69° + k \cdot 360°$; $k \in \mathbb{Z}$